Journal of Fuzzy Extension and Applications

www.journal-fea.com

J. Fuzzy. Ext. Appl. Vol. 6, No. 1 (2025) 43–58.

Paper Type: Research Paper

Soft Intersection Almost Bi-Quasi-Interior Ideals of Semigroups

Aslihan Sezgin[1,*], Zeynep Hare Baş[2], Aleyna Ilgin[2]

[1] Department of Mathematics and Science Education, Faculty of Education, Amasya University, Amasya, Turkey; aslihan.sezgin@amasya.edu.tr.

[2] Department of Mathematics, Graduate School of Natural and Applied Sciences, Amasya University, Amasya, Turkey; zeynephare05@gmail.com; aleynailgiin@gmail.com.

Citation:

Received: 16 April 2024 *Revised: 15 June 2024* *Accepted: 23 August 2024*	Sezgin, A., Baş, Z. H, & İlgin, A. (2025). Soft intersection almost bi-quasi-interior ideals of semigroups. *Journal of fuzzy extension and applications, 6*(1), 43-58.

Abstract

Generalizing the ideals of an algebraic structure has shown to be both exciting and valuable to mathematicians. In this context, the concept of Bi-Quasi-Interior Ideal (BQI-ideal) was proposed as a generalization of interior ideal, quasi-ideal, bi-ideal, bi-quasi ideal, and bi-interior ideal in a semigroup in a manner similar to how the notion of soft intersection BQI-ideal was presented to generalize the soft intersection interior ideal, quasi-ideal, bi-ideal, bi-quasi ideal, and bi-interior ideal in a semigroup. In this study, we introduce new types of soft intersection almost ideals called "soft intersection (weakly) almost BQI-ideal" and discuss the concepts in terms of their algebraic structures, providing some characterizations regarding the concepts. To explain the essential properties of these ideals, we employ algebraic methods. The primary objective of this paper is to establish the relationship between soft intersection (weakly) almost BQI-ideals and other certain types of almost ideals in semigroups and soft intersection ideals. It is obtained that a nonnull soft intersection BQI-ideal is a soft intersection almost BQI-ideal and that a soft intersection almost BQI-ideal is a soft intersection weakly almost BQI-ideal; however, the counterparts are not valid with counterexamples. Additionally, we obtain that any idempotent soft intersection almost BQI-ideal is a soft intersection almost subsemigroup. With our obtained crucial theorem that states that if a nonempty set of a semigroup is an almost BQI-ideal, then its soft characteristic function is a soft intersection almost BQI-ideal, and vice versa, we achieve to construct a bridge between semigroup theory and soft set theory. With this vital theorem, several interesting relationships between certain types of almost BQI-ideals of semigroups, such as minimal, prime, semiprime, and strongly prime almost BQI-ideals and certain types of soft intersection almost BQI-ideals, are derived. Besides, we support our assertions with illustrative and concrete examples.

Keywords: Soft set, Almost Bi-quasi-interior ideals, Soft intersection almost Bi-quasi-interior ideals, Semigroup.

1 | Introduction

The abstract algebraic basis for "memoryless" systems restart on each iteration provided by semigroups, which makes them essential to many areas of mathematics. Semigroups, formally studied in the early 1900s, are

Corresponding Author: aslihan.sezgin@amasya.edu.tr

https://doi.org/10.22105/jfea.2024.452790.1445

crucial models for linear time-invariant systems in practical mathematics. Studying finite semigroups is crucial to theoretical computer science because they are intrinsically linked to finite automata. Moreover, semigroups and Markov processes are connected in probability theory. To comprehend mathematical structures, the concept of ideals is of great importance, and generalizing ideals in algebraic structures has been an important area of study for many mathematicians. Bi-ideals for semigroups were proposed for the first time in [1]. The concept of quasi-ideals was introduced for semigroups in [2], and it was later extended to rings. The idea of almost ideals of semigroups was first proposed in [3]. The notion of bi-ideals was expanded to almost bi-ideals of semigroups later in [4]. While the concept of almost quasi-ideals was proposed in [5], the concepts of almost ideals and interior ideals of semigroups were extended and analyzed by introducing almost interior ideals and weakly almost interior ideals of semigroups in [6]. In [7–10], the authors presented almost subsemigroups, almost Bi-Quasi-Interior Ideals (BQI-ideals), almost bi-interior ideals, and almost bi-quasi-ideals of semigroups, respectively. Additionally, several almost fuzzy semigroup ideal types were examined in [5], [7–12].

The concept of soft sets as a means of defining uncertainty was initially presented by Molodtsov [13] in 1999, and it has since attracted interest from several academic disciplines. In [14–29], the fundamental operations of soft sets were examined. The concepts of soft sets and soft set operations were modified by Çağman and Enginoğlu [30], and Çağman et al. proposed soft intersection groups in [31], which served as an inspiration for research on additional soft algebraic systems. Soft sets were thoroughly and firstly conveyed to semigroup theory in [32–34] by defining certain classes of soft intersection ideals (from now on denoted by SI-ideals) of semigroups. Further research was conducted on various soft algebraic structures in [35–44]. Rao [45–48] has developed several novel new types of ideals of semigroup, which are generalizations of the ones that already exist. Furthermore, Baupradist et al. proposed the idea of essential ideals in semigroups [49]. Rao [46] introduced the BQI-ideal of semigroups as a generalization [47] of bi-ideal, quasi-ideal, interior ideal, bi-interior ideal, and bi-quasi ideal of semigroups, and SI-BQI-ideal of semigroups was introduced in this research with the similar aim.

In this paper, we propose to construct a more comprehensive generalization with the aid of SI-(weakly) almost BQI-ideal and to establish the relationship among SI-BQI-ideal and other classes of SI-ideals and almost ideals of semigroups, as the generalizations of SI-ideals have been a fast developing subject of research for many years, We were motivated and inspired by the algebraic structures of SI-BQI-ideal studied in . In order to accomplish this, we delve into the notion of SI-almost BQI-ideal and its generalization SI-weakly almost BQI-ideal, which are presented as a further generalization of the nonnull SI-BQI-ideal. We show that an idempotent SI-almost BQI-ideal is also an SI-almost subsemigroup. We also obtain that a semigroup may be formed by SI-almost BQI-ideal with the union operation of soft sets but not with the intersection operation of soft sets, unlike from the classical semigroup theory. Furthermore, we obtain the relations between minimal (resp., prime, semiprime, strongly prime) almost BQI-ideal of a semigroup and minimal (resp., prime, semiprime, strongly prime) SI-almost BQI-ideal. This is accomplished by establishing that the soft characteristic function of a nonempty set of a semigroup is an SI-almost BQI-ideal if and only if the set is an almost BQI-ideal, and vice versa. With this crucial theorem, we also succeed in building a connection between soft set theory and classical semigroup theory. The paper is divided into four sections. A summary of the subject is given in Section 1, and an investigation of the ideals of semigroups and soft sets and their pertinent definitions and outcomes are covered in Section 2. In Section 3, we discuss the properties of the SI-(weakly) almost BQI-ideals of semigroups together with their relations with other certain types of SI-ideals and ideals in semigroups, providing concrete examples. In Section 4, we wrap up our investigation together with possible future studies.

2 | Preliminaries

Definition 1 ([13], [30]). Let E be the parameter set, U be the universal set, $P(U)$ be the power set of U, and $\Upsilon \subseteq E$. The soft set f_Υ over U is a function such that $f_\Upsilon: E \rightarrow P(U)$, where for all $x \notin \Upsilon$, $f_\Upsilon(x) = \emptyset$. That is,

$$f_Y = \{(x, f_Y(x)): x \in E, f_Y(x) \in P(U)\}.$$

The set of all soft sets over U is designated by $S_E(U)$ throughout this paper.

Definition 2 ([30]). Let $f_{\mathcal{H}} \in S_E(U)$. If $f_{\mathcal{H}}(x) = \emptyset$ for all $x \in E$, then $f_{\mathcal{H}}$ is called a null soft set and indicated by $\emptyset_E$.

Definition 3 ([30]). Let $f_{\mathcal{H}}, f_{\aleph} \in S_E(U)$. If $f_{\mathcal{H}}(x) \subseteq f_{\aleph}(x)$, for all $x \in E$, then $f_{\mathcal{H}}$ is a soft subset of $f_{\aleph}$ and indicated by $f_{\mathcal{H}} \tilde{\subseteq} f_{\aleph}$. If $f_{\mathcal{H}}(x) = f_{\aleph}(x)$, for all $x \in E$, then $f_{\mathcal{H}}$ is called soft, equal to $f_{\aleph}$ and denoted by $f_{\mathcal{H}} = f_{\aleph}$.

Definition 4 ([30]). Let $f_{\mathcal{H}}, f_{\aleph} \in S_E(U)$. The union (intersection) of $f_{\mathcal{H}}$ and $f_{\aleph}$ is the soft set $f_{\mathcal{H}} \tilde{\cup} f_{\aleph}$ ($f_{\mathcal{H}} \tilde{\cap} f_{\aleph}$) where $(f_{\mathcal{H}} \tilde{\cup} f_{\aleph})(x) = f_{\mathcal{H}}(x) \cup f_{\aleph}(x)$ $\left((f_{\mathcal{H}} \tilde{\cap} f_{\aleph})(x) = f_{\mathcal{H}}(x) \cap f_{\aleph}(x)\right)$, for all $x \in E$.

Definition 5 ([24]). Let $t_A \in S_E(U)$. Then,

$$\operatorname{supp}(t_A) = \{x \in A: t_A(x) \neq \emptyset\}.$$

Is called the support of t_A. A soft set with an empty support is a null soft set; otherwise, the soft set is non-null.

Note 1: if $t_A \tilde{\subseteq} k_B$, then $\operatorname{supp}(t_A) \subseteq \operatorname{supp}(k_B)$ [50].

S stands for a semigroup throughout this paper. Let $\emptyset \neq H \subseteq S$. Then, H is called a bi-quasi-interior (BQI-ideal) of S if $HSHSH \subseteq H$ [46], is called an almost BQI-ideal of S if $xHyH \cap H \neq \emptyset$, for all $x, y \in S$ [8]. An almost BQI-ideal H of S is called a minimal almost BQI-ideal of S if for any almost BQI-ideal B of S if whenever $B \subseteq H$, then $H = B$. An almost BQI-ideal $\wp$ of S is called a prime almost BQI-ideal if for any almost BQI-ideals H and B of S such that $HB \subseteq \wp$ implies that $H \subseteq \wp$ or $B \subseteq \wp$. An almost BQI-ideal $\wp$ of S is called a semiprime almost BQI-ideal if for any almost BQI-ideal H of S such that $HH \subseteq \wp$ implies that $H \subseteq \wp$. An almost BQI-ideal $\wp$ of S is called a strongly prime almost BQI-ideal if for any almost BQI-ideals H and B of S such that $HB \cap BH \subseteq \wp$ implies that $H \subseteq \wp$ or $B \subseteq \wp$ [8].

Definition 6. Let $f_S, g_S \in S_S(U)$ [32]. Soft intersection product $f_S \circ g_S$ is defined by

$$(f_S \circ g_S)(x) = \begin{cases} \bigcup\limits_{x=yz,} \{f_S(y) \cap g_S(z)\}, & \text{if there existsy, } z \in S \text{ such that } x = yz, \\ \emptyset, & \text{otherwise.} \end{cases}$$

Theorem 1. Let $p_S, \varkappa_S, \vartheta_S \in S_S(U)$ [32]. Then,

I. $(p_S \circ \varkappa_S) \circ \vartheta_S = p_S \circ (\varkappa_S \circ \vartheta_S)$.

II. $p_S \circ \varkappa_S \neq p_S \circ \varkappa_S$.

III. $p_S \circ (\varkappa_S \tilde{\cup} \vartheta_S) = (p_S \circ \varkappa_S) \tilde{\cup} (p_S \circ \vartheta_S)$ and $(p_S \tilde{\cup} \varkappa_S) \circ \vartheta_S = (p_S \circ \vartheta_S) \tilde{\cup} (\varkappa_S \circ \vartheta_S)$.

IV. $p_S \circ (\varkappa_S \tilde{\cap} \vartheta_S) = (p_S \circ \varkappa_S) \tilde{\cap} (p_S \circ \vartheta_S)$ and $(p_S \tilde{\cap} \varkappa_S) \circ \vartheta_S = (p_S \circ \vartheta_S) \tilde{\cap} (\varkappa_S \circ \vartheta_S)$.

V. If $p_S \tilde{\subseteq} \varkappa$, then $p_S \circ t_S \tilde{\subseteq} \varkappa_S \circ t_S$ and $t_S \circ p_S \tilde{\subseteq} t_S \circ \varkappa_S$.

VI. If $\mathfrak{H}_S, y_S \in S_S(U)$ such that $\mathfrak{H}_S \tilde{\subseteq} p_S$ and $y_S \tilde{\subseteq} q_S$, then $\mathfrak{H}_S \circ y_S \tilde{\subseteq} p_S \circ q_S$.

Lemma 1. Let $f_S, g_S \in S_S(U)$. Then, $f_S \circ g_S = \emptyset_S$ if and only if $f_S = \emptyset_S$ or $g_S = \emptyset_S$ [52].

Definition 7 ([32]). Let $A \subseteq S$. The soft characteristic function of A, denoted by S_A, is defined as

$$S_A(x) = \begin{cases} U, & \text{if } x \in A, \\ \emptyset, & \text{if } x \in S \backslash A. \end{cases}$$

It is obvious that $S_A: S \longrightarrow P(U)$.

Corollary 1 ([50]). $\operatorname{supp}(S_A) = A$.

Theorem 2 ([32], [50]). Let $\mathcal{H}, \mathcal{M} \subseteq S$. Then,

I. $\mathcal{H} \subseteq \mathcal{M}$ if and only if $S_{\mathcal{H}} \tilde{\subseteq} S_{\mathcal{M}}$.

II. $S_{\mathcal{H}} \tilde{\cap} S_{\mathcal{M}} = S_{\mathcal{H}\cap\mathcal{M}}$ and $S_{\mathcal{H}} \tilde{\cup} S_{\mathcal{M}} = S_{\mathcal{H}\cup\mathcal{M}}$.

III. $S_{\mathcal{H}} \circ S_{\mathcal{M}} = S_{\mathcal{H}\mathcal{M}}$.

Definition 8 ([51]). The soft characteristic function of x, denoted by S_x, is defined as

$$S_x(y) = \begin{cases} U, & \text{if } y = x, \\ \emptyset, & \text{if } y \neq x. \end{cases}$$

where $x \in S$. It is clear that $S_x: S \longrightarrow P(U)$.

Corollary 2. Let $x, y \in S$, and $f_S, S_x \in S_S(U)$. Then, $f_S \circ S_x \circ f_S \circ S_y \circ f_S = \emptyset_S$ if and only if $f_S = \emptyset_S$.

Proof: By *Lemma 1*, $f_S \circ S_x \circ f_S \circ S_y \circ f_S = \emptyset_S$ if and only if $f_S = \emptyset_S$ or $S_x = \emptyset_S$ or $S_y = \emptyset_S$. Since $S_x \neq \emptyset_S$ and $S_y \neq \emptyset_S$, for all $x \in S$ by *Definition 8*, $f_S = \emptyset_S$. The rest is obvious by *Lemma 1*.

Definition 9. A soft set f_S is called an SI-BQI-ideal of S over U if $f_S(xyztw) \supseteq f_S(x) \cap f_S(z) \cap f_S(w)$, for all $x, y, z, t, w \in S$.

If $f_S(x) = U$ for all $x \in S$, then f_S is an SI-BQI-ideal of S, which is indicated by $\tilde{\mathbb{S}}$. Note that $\tilde{\mathbb{S}} = S_S$, i.e., $\tilde{\mathbb{S}}(x) = U$, for all $x \in S$.

Theorem 3. The soft set f_S is an SI-BQI-ideal of S if and only if $f_S \circ \tilde{\mathbb{S}} \circ f_S \circ \tilde{\mathbb{S}} \circ f_S \tilde{\subseteq} f_S$.

Definition 10 ([50]). A soft set is called an SI-almost subsemigroup of S if $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$.

Regarding the potential consequences of network analysis and graph applications on soft sets-which are determined by the divisibility of determinants-we refer to [52]; for more about the operations of soft sets, we refer to [53], [54], and for more about Intuitionistic, Pythagorean, Picture, and Hesitant fuzzy sets, along with Neutrosophic, Plithogenic, and Hypersoft sets, we refer to [55–63].

3 | Soft Intersection Almost BQI-Ideal of Semigroups

Definition 11. A soft set f_S is called a soft intersection, an almost BQI-ideal of S if

$$\left(f_S \circ S_x \circ f_S \circ S_y \circ f_S\right) \tilde{\cap} f_S \neq \emptyset_S.$$

for all $x, y \in S$, and f_S is called a soft intersection weakly almost BQI-ideal of S if

$$(f_S \circ S_x \circ f_S \circ S_x \circ f_S) \tilde{\cap} f_S \neq \emptyset_S.$$

for all $x \in S$. The term soft intersection is abbreviated as "SI"; bi-quasi-interior is as "BQI"; thus, soft intersection (weakly) almost BQI-ideal of S is indicated by "SI-(weakly) almost BQI-ideal" from now on.

Example 1. Let S={♎, ♉} be given with the following *Table 1*.

Table 1. Cayley table of the binary operation.

.	♎	♉
♎	♎	♉
♉	♉	♎

Then, (S, .) is a semigroup. Let f_s, l_s, and g_s over $U = \left\{\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} \mid a \in \mathbb{Q}'\right\}$ as follows:

$$f_S = \left\{\left(♎, \left\{\begin{bmatrix} e & 0 \\ 0 & e \end{bmatrix}, \begin{bmatrix} e^2 & 0 \\ 0 & e^2 \end{bmatrix}\right\}\right), \left(♉, \left\{\begin{bmatrix} e & 0 \\ 0 & e \end{bmatrix}, \begin{bmatrix} e^3 & 0 \\ 0 & e^3 \end{bmatrix}\right\}\right)\right\}.$$

$$l_S = \left\{\left(♎, \left\{\begin{bmatrix} e^4 & 0 \\ 0 & e^4 \end{bmatrix}\right\}\right), \left(♉, \left\{\begin{bmatrix} e^4 & 0 \\ 0 & e^4 \end{bmatrix}, \begin{bmatrix} e^2 & 0 \\ 0 & e^2 \end{bmatrix}\right\}\right)\right\}.$$

$$g_S = \left\{\left(\varrho, \left\{\begin{bmatrix} e^3 & 0 \\ 0 & e^3 \end{bmatrix}, \begin{bmatrix} e^4 & 0 \\ 0 & e^4 \end{bmatrix}\right\}\right), \left(\gamma, \left\{\begin{bmatrix} e & 0 \\ 0 & e \end{bmatrix}\right\}\right)\right\}.$$

Here, f_S and l_S are both SI-almost BQI-ideals, while g_S is not an SI-almost BQI-ideal. Let's first show that f_S is an SI-almost BQI ideal, that is,

$$\left(f_S \circ S_x \circ f_S \circ S_y \circ f_S\right) \widetilde{\cap}\, f_S \neq \emptyset_S.$$

for all $x, y \in S$. Let's start with $S_\varrho - S_\varrho$:

$$\Big[\big(f_S \circ S_\varrho \circ f_S \circ S_\varrho \circ f_S\big) \widetilde{\cap}\, f_S\Big](\varrho) = \Big[\Big(\big(f_S \circ S_\varrho\big)(\varrho) \cap (f_S \circ S_\varrho \circ f_S)(\varrho)\Big) \cup \Big(\big(f_S \circ S_\varrho\big)(\gamma) \cap$$
$$(f_S \circ S_\varrho \circ f_S)(\gamma)\Big)\Big] \cap f_S(\varrho) = \Big[\Big(f_S(\varrho) \cap S_\varrho(\varrho)\Big) \cup \Big(f_S(\gamma) \cap S_\varrho(\gamma)\Big) \cap \Big(f_S \circ S_\varrho\big)(\varrho) \cap$$
$$f_S(\varrho) \cup (f_S \circ S_\varrho)(\gamma) \cap f_S(\gamma)\Big) \cup \Big(\Big(f_S(\varrho) \cap S_\varrho(\varrho)\Big) \cup \Big(f_S(\gamma) \cap S_\varrho(\varrho)\Big) \cap$$
$$\Big(f_S \circ S_\varrho)(\varrho) \cap f_S(\gamma) \cup (f_S \circ S_\varrho)(\gamma) \cap f_S(\varrho)\Big)\Big)\Big] \cap f_S(\varrho) = \Big[f_S(\varrho) \cap \Big[\Big(f_S(\varrho) \cap S_\varrho(\varrho)) \cup$$
$$(f_S(\gamma) \cap S_\varrho(\gamma)\Big)] \cap f_S(\varrho) \cup \Big[\Big(f_S(\varrho) \cap S_\varrho(\gamma)\Big) \cup \Big(f_S(\gamma) \cap S_\varrho(\varrho)\Big)\Big] \cap f_S(\gamma) \cup$$
$$\Big(f_S(\gamma) \cap [(f_S(\varrho) \cap S_\varrho(\varrho)\Big) \cup (f_S(\gamma) \cap S_\varrho(\gamma)\Big] \cap f_S(\gamma) \cup \Big[\Big(f_S(\varrho) \cap S_\varrho(\gamma)\Big) \cup$$
$$\Big(f_S(\gamma) \cap S_\varrho(\varrho)\Big)\Big] \cap f_S(\varrho)\Big] \cap f_S(\varrho) = [[f_S(\varrho) \cap (f_S(\varrho)] \cap f_S(\varrho) \cup f_S(\gamma) \cap f_S(\gamma) \cup$$
$$f_S(\gamma) \cap f_S(\varrho) \cap f_S(\gamma) \cup f_S(\gamma) \cap f_S(\varrho)\Big] \cap f_S(\varrho) = f_S(\varrho).$$

$$\begin{aligned}
&\Big[\big(f_S \circ S_\varrho \circ f_S \circ S_\varrho \circ f_S\big) \widetilde{\cap}\, f_S\Big](\gamma) \\
&\qquad = \Big[\Big(\big(f_S \circ S_\varrho\big)(\varrho) \cap (f_S \circ S_\varrho \circ f_S)(\gamma)\Big) \\
&\qquad \cup \Big(\big(f_S \circ S_\varrho\big)(\gamma) \cap \big(f_S \circ S_\varrho \circ f_S\big)(\varrho)\Big)\Big] \cap f_S(\gamma) \\
&\qquad = \Big[\Big(f_S(\varrho) \cap \big(f_S(\varrho) \cap f_S(\gamma)\big)\Big) \cup \Big(f_S(\gamma) \cap \big((f_S(\varrho) \cup f_S(\gamma)\big)\Big)\Big] \cap f_S(\gamma) \\
&\qquad = \Big[\big(f_S(\varrho) \cap f_S(\gamma)\big) \cup f_S(\gamma)\Big] \cap f_S(\gamma) = f_S(\gamma) \cap f_S(\gamma) = f_S(\gamma).
\end{aligned}$$

Thus, $\left(f_S \circ S_\varrho \circ f_S \circ S_\varrho \circ f_S\right) \widetilde{\cap}\, f_S = \left\{\left(\varrho, \left\{\begin{bmatrix} e & 0 \\ 0 & e \end{bmatrix}, \begin{bmatrix} e^2 & 0 \\ 0 & e^2 \end{bmatrix}\right\}\right), \left(\gamma, \left\{\begin{bmatrix} e & 0 \\ 0 & e \end{bmatrix}, \begin{bmatrix} e^3 & 0 \\ 0 & e^3 \end{bmatrix}\right\}\right)\right\} \neq \emptyset.$

Let's continue with $S_\gamma - S_\gamma$;

$$\begin{aligned}
&[(f_S \circ S_\gamma \circ f_S \circ S_\gamma \circ f_S) \widetilde{\cap}\, f_S](\varrho) \\
&\qquad = \big[\big((f_S \circ S_\gamma)(\varrho) \cap (f_S \circ S_\gamma \circ f_S)(\varrho)\big) \cup \big((f_S \circ S_\gamma)(\gamma) \cap (f_S \circ S_\gamma \circ f_S)(\gamma)\big)\big] \\
&\qquad \cap f_S(\varrho) \\
&\qquad = \Big[\Big(f_S(\gamma) \cap \big(f_S(\varrho) \cap f_S(\gamma)\big)\Big) \cup \Big(f_S(\varrho) \cap \big(f_S(\varrho) \cup f_S(\gamma)\big)\Big)\Big] \cap f_S(\varrho) \\
&\qquad = \Big[\big(f_S(\varrho) \cap f_S(\gamma)\big) \cup f_S(\varrho)\Big] \cap f_S(\varrho) = f_S(\varrho) \cap f_S(\varrho) = f_S(\varrho).
\end{aligned}$$

$$\begin{aligned}
&[(f_S \circ S_\gamma \circ f_S \circ S_\gamma \circ f_S) \widetilde{\cap}\, f_S](\gamma) \\
&\qquad = \big[\big((f_S \circ S_\gamma)(\varrho) \cap (f_S \circ S_\gamma \circ f_S)(\gamma)\big) \cup \big((f_S \circ S_\gamma)(\gamma) \cap (f_S \circ S_\gamma \circ f_S)(\varrho)\big)\big] \\
&\qquad \cap f_S(\gamma) \\
&\qquad = \Big[\Big(f_S(\gamma) \cap \big(f_S(\varrho) \cup f_S(\gamma)\big)\Big) \cup \Big(f_S(\varrho) \cap \big(f_S(\varrho) \cap f_S(\gamma)\big)\Big)\Big] \cap f_S(\gamma) \\
&\qquad = \Big[f_S(\gamma) \cup \big(f_S(\varrho) \cap f_S(\gamma)\big)\Big] \cap f_S(\gamma) = f_S(\gamma) \cap f_S(\gamma) = f_S(\gamma).
\end{aligned}$$

Thus, $(f_S \circ S_\gamma \circ f_S \circ S_\gamma \circ f_S) \widetilde{\cap}\, f_S = \left\{\left(\varrho, \left\{\begin{bmatrix} e & 0 \\ 0 & e \end{bmatrix}, \begin{bmatrix} e^2 & 0 \\ 0 & e^2 \end{bmatrix}\right\}\right), \left(\gamma, \left\{\begin{bmatrix} e & 0 \\ 0 & e \end{bmatrix}, \begin{bmatrix} e^3 & 0 \\ 0 & e^3 \end{bmatrix}\right\}\right)\right\} \neq \emptyset.$

Let's continue with $S_\varrho - S_\gamma$:

$$\begin{aligned}
&[(f_S \circ S_ջ \circ f_S \circ S_ɤ \circ f_S) \tilde{\cap} f_S](ջ) \\
&\qquad = \Big[\big((f_S \circ S_ջ)(ջ) \cap (f_S \circ S_ɤ \circ f_S)(ջ)\big) \cup \big((f_S \circ S_ջ)(ɤ) \cap (f_S \circ S_ɤ \circ f_S)(ɤ)\big)\Big] \cap f_S(ջ \\
&\qquad = \Big[\big(f_S(ջ) \cap (f_S(ջ) \cap f_S(ɤ))\big) \cup \big(f_S(ɤ) \cap (f_S(ջ) \cup f_S(ɤ))\big)\Big] \cap f_S(ջ) \\
&\qquad = [(f_S(ջ) \cap f_S(ɤ)) \cup f_S(ɤ)] \cap f_S(ջ) = f_S(ɤ) \cap f_S(ջ)
\end{aligned}$$

$$\begin{aligned}
&[(f_S \circ S_ջ \circ f_S \circ S_ɤ \circ f_S) \tilde{\cap} f_S](ɤ) \\
&\qquad = \Big[\big((f_S \circ S_ջ)(ջ) \cap (f_S \circ S_ɤ \circ f_S)(ɤ)\big) \cup \big((f_S \circ S_ջ)(ɤ) \cap (f_S \circ S_ɤ \circ f_S)(ջ)\big)\Big] \cap f_S(ɤ \\
&\qquad = \Big[\big(f_S(ջ) \cap (f_S(ջ) \cup f_S(ɤ))\big) \cup \big(f_S(ɤ) \cap (f_S(ջ) \cap f_S(ɤ))\big)\Big] \cap f_S(ɤ) \\
&\qquad = [f_S(ջ) \cup (f_S(ɤ) \cap f_S(ջ))] \cap f_S(ɤ) = f_S(ջ) \cap f_S(ɤ)
\end{aligned}$$

Therefore, $(f_S \circ S_ջ \circ f_S \circ S_ɤ \circ f_S) \tilde{\cap} f_S = \left\{\left(ջ, \left\{\begin{bmatrix} e & 0 \\ 0 & e \end{bmatrix}\right\}\right), \left(ɤ, \left\{\begin{bmatrix} e & 0 \\ 0 & e \end{bmatrix}\right\}\right)\right\} \neq \emptyset$.

Let's continue with $S_ɤ - S_ջ$:

$$\begin{aligned}
&[(f_S \circ S_ɤ \circ f_S \circ S_ջ \circ f_S) \tilde{\cap} f_S](ջ) \\
&\qquad = \Big[\big((f_S \circ S_ɤ)(ջ) \cap (f_S \circ S_ջ \circ f_S)(ջ)\big) \cup \big((f_S \circ S_ɤ)(ɤ) \cap (f_S \circ S_ջ \circ f_S)(ɤ)\big)\Big] \\
&\qquad \cap f_S(ջ) = \Big[\big(f_S(ɤ) \cap (f_S(ջ) \cup f_S(ɤ))\big) \cup \big(f_S(ջ) \cap ((f_S(ջ) \cap f_S(ɤ))\big)\Big] \cap f_S(ջ) \\
&\qquad = [f_S(ɤ) \cup (f_S(ջ) \cap f_S(ɤ))] \cap f_S(ջ) = f_S(ɤ) \cap f_S(ջ).
\end{aligned}$$

$$\begin{aligned}
&[(f_S \circ S_ɤ \circ f_S \circ S_ջ \circ f_S) \tilde{\cap} f_S](ɤ) \\
&\qquad = \Big[\big((f_S \circ S_ɤ)(ջ) \cap (f_S \circ S_ջ \circ f_S)(ɤ)\big) \cup \big((f_S \circ S_ɤ)(ɤ) \cap (f_S \circ S_ջ \circ f_S)(ջ)\big)\Big] \\
&\qquad \cap f_S(ɤ) = \Big[\big(f_S(ɤ) \cap (f_S(ջ) \cap f_S(ɤ))\big) \cup \big(f_S(ջ) \cap ((f_S(ջ) \cup f_S(ɤ))\big)\Big] \cap f_S(ɤ) \\
&\qquad = [(f_S(ɤ) \cap f_S(ջ)) \cup f_S(ջ)] \cap f_S(ɤ) = f_S(ջ) \cap f_S(ɤ) = \emptyset.
\end{aligned}$$

Accordingly, $(f_S \circ S_ɤ \circ f_S \circ S_ջ \circ f_S) \tilde{\cap} f_S = \left\{\left(ջ, \left\{\begin{bmatrix} e & 0 \\ 0 & e \end{bmatrix}\right\}\right), \left(ɤ, \left\{\begin{bmatrix} e & 0 \\ 0 & e \end{bmatrix}\right\}\right)\right\} \neq \emptyset$.

Therefore, $(f_S \circ S_x \circ f_S \circ S_y \circ f_S) \tilde{\cap} f_S \neq \emptyset_s$, for all $x, y \in S$ implying that f_s is an SI-almost BQI-ideal. Similarly, l_s is an SI-almost BQI-ideal. In fact,

$$(l_s \circ S_ջ \circ l_s \circ S_ջ \circ l_s) \tilde{\cap} l_s = \left\{\left(ջ, \left\{\begin{bmatrix} e^4 & 0 \\ 0 & e^4 \end{bmatrix}\right\}\right), \left(ɤ, \left\{\begin{bmatrix} e^4 & 0 \\ 0 & e^4 \end{bmatrix}, \begin{bmatrix} e^2 & 0 \\ 0 & e^2 \end{bmatrix}\right\}\right)\right\} \neq \emptyset.$$

$$(l_s \circ S_ջ \circ l_s \circ S_ɤ \circ l_s) \tilde{\cap} l_s = \left\{\left(ջ, \left\{\begin{bmatrix} e^4 & 0 \\ 0 & e^4 \end{bmatrix}\right\}\right), \left(ɤ, \left\{\begin{bmatrix} e^4 & 0 \\ 0 & e^4 \end{bmatrix}\right\}\right)\right\} \neq \emptyset.$$

$$(l_s \circ S_ɤ \circ l_s \circ S_ջ \circ l_s) \tilde{\cap} l_s = \left\{\left(ջ, \left\{\begin{bmatrix} e^4 & 0 \\ 0 & e^4 \end{bmatrix}\right\}\right), \left(ɤ, \left\{\begin{bmatrix} e^4 & 0 \\ 0 & e^4 \end{bmatrix}\right\}\right)\right\} \neq \emptyset.$$

$$(l_s \circ S_ɤ \circ l_s \circ S_ɤ \circ l_s) \tilde{\cap} l_s = \left\{\left(ջ, \left\{\begin{bmatrix} e^4 & 0 \\ 0 & e^4 \end{bmatrix}\right\}\right), \left(ɤ, \left\{\begin{bmatrix} e^4 & 0 \\ 0 & e^4 \end{bmatrix}, \begin{bmatrix} e^2 & 0 \\ 0 & e^2 \end{bmatrix}\right\}\right)\right\} \neq \emptyset.$$

One can also show that g_S is not an SI-almost BQI-ideal. In fact,

$$\begin{aligned}
&[(g_s \circ S_ջ \circ g_s \circ S_ɤ \circ g_s) \tilde{\cap} g_s](ջ) \\
&\qquad = \Big[\big((g_s \circ S_ջ)(ջ) \cap (g_s \circ S_ɤ \circ g_s)(ջ)\big) \cup \big((g_s \circ S_ջ)(ɤ) \cap (g_s \circ S_ɤ \circ g_s)(ɤ)\big)\Big] \\
&\qquad \cap g_s(ջ) = \Big[\big(g_s(ջ) \cap (g_s(ջ) \cap g_s(ɤ))\big) \cup \big(g_s(ɤ) \cap (g_s(ջ) \cup g_s(ɤ))\big)\Big] \cap g_s(ջ) \\
&\qquad = [(g_s(ջ) \cap g_s(ɤ)) \cup g_s(ɤ)] \cap g_s(ջ) = g_s(ɤ) \cap g_s(ջ) = \emptyset.
\end{aligned}$$

$$
\begin{aligned}
&[(g_s \circ S_\varrho \circ g_s \circ S_\gamma \circ g_s) \tilde{\cap} g_s](\gamma) \\
&\qquad = \Big[\Big((g_s \circ S_\varrho)(\varrho) \cap (g_s \circ S_\gamma \circ g_s)(\gamma)\Big) \cup \Big((g_s \circ S_\varrho)(\gamma) \cap (g_s \circ S_\gamma \circ g_s)(\varrho)\Big)\Big] \\
&\qquad \cap g_s(\gamma) = \Big[\Big(g_s(\varrho) \cap \big(g_s(\varrho) \cup g_s(\gamma)\big)\Big) \cup \Big(g_s(\gamma) \cap \big(g_s(\varrho) \cap g_s(\gamma)\big)\Big)\Big] \cap g_s(\gamma) \\
&\qquad = [g_s(\varrho) \cup (g_s(\gamma) \cap g_s(\varrho))] \cap g_s(\gamma) = g_s(\varrho) \cap g_s(\gamma) = \emptyset.
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
&[(g_s \circ S_\gamma \circ g_s \circ S_\varrho \circ g_s) \tilde{\cap} g_s](\varrho) \\
&\qquad = \Big[\Big((g_s \circ S_\gamma)(\varrho) \cap (g_s \circ S_\varrho \circ g_s)(\varrho)\Big) \cup \Big((g_s \circ S_\gamma)(\gamma) \cap (g_s \circ S_\varrho \circ g_s)(\gamma)\Big)\Big] \\
&\qquad \cap g_s(\varrho) = \Big[\Big(g_s(\gamma) \cap \big(g_s(\varrho) \cup g_s(\gamma)\big)\Big) \cup \Big(g_s(\varrho) \cap \big((g_s(\varrho) \cap g_s(\gamma)\big)\Big)\Big] \cap g_s(\varrho) \\
&\qquad = [g_s(\gamma) \cup (g_s(\varrho) \cap g_s(\gamma))] \cap g_s(\varrho) = g_s(\gamma) \cap g_s(\varrho) = \emptyset.
\end{aligned}
$$

$$
\begin{aligned}
&[(g_s \circ S_\gamma \circ g_s \circ S_\varrho \circ g_s) \tilde{\cap} g_s](\gamma) \\
&\qquad = \Big[\Big((g_s \circ S_\gamma)(\varrho) \cap (g_s \circ S_\varrho \circ g_s)(\gamma)\Big) \cup \Big((g_s \circ S_\gamma)(\gamma) \cap (g_s \circ S_\varrho \circ g_s)(\varrho)\Big)\Big] \\
&\qquad \cap g_s(\gamma) \\
&\qquad = \Big[\Big(g_s(\gamma) \cap \big(g_s(\varrho) \cap g_s(\gamma)\big)\Big) \cup \Big(g_s(\varrho) \cap \big((g_s(\varrho) \cup g_s(\gamma)\big)\Big)\Big] \cap g_s(\gamma) \\
&\qquad = [(g_s(\gamma) \cap g_s(\varrho)) \cup g_s(\varrho)] \cap g_s(\gamma) = g_s(\varrho) \cap g_s(\gamma) = \emptyset.
\end{aligned}
$$

Accordingly, $(g_s \circ S_x \circ g_s \circ S_y \circ g_s) \tilde{\cap} g_s = \emptyset_s$, for some $x, y \in S$. Thus, g_s is not an SI-almost BQI-ideal.

Proposition 1. Every SI-almost BQI-ideal is an SI-weakly almost BQI-ideal.

Proof: assume that t_S is an SI-almost BQI-ideal of S. Then, $(t_S \circ S_x \circ t_S \circ S_y \circ t_S)$, for all $x, y \in S$. Hence, $(t_S \circ S_x \circ t_S \circ S_x \circ t_S) \tilde{\cap} t_S \neq \emptyset_S$, for all $x \in S$. Thus, t_S is an SI-weakly almost BQI-ideal.

Proposition 1 shows that SI-weakly almost BQI-ideal is a generalization of SI-almost BQI-ideal. Here, note that if t_S is an SI-weakly almost BQI-ideal, then it need not be an SI-almost BQI-ideal as shown with the following example:

Example 2. The soft set g_s in *Example 1* is an SI-weakly almost BQI-ideal; however, it is not an SI-almost BQI-ideal. In fact,

$$
\begin{aligned}
&[(g_s \circ S_\varrho \circ g_s \circ S_\varrho \circ g_s) \tilde{\cap} g_s](\varrho) = \Big[\Big((g_s \circ S_\varrho)(\varrho) \cap (g_s \circ S_\varrho \circ g_s)(\varrho)\Big) \cup \Big((g_s \circ S_\varrho)(\gamma) \cap \\
&(g_s \circ S_\varrho \circ g_s)(\gamma)\Big)\Big] \cap g_s(\varrho) = \Big[\Big(g_s(\varrho) \cap \big(g_s(\varrho) \cup g_s(\gamma)\big)\Big) \cup \Big(g_s(\gamma) \cap \big(g_s(\gamma) \cap g_s(\varrho)\big)\Big)\Big] \cap \\
&g_s(\varrho) = [g_s(\varrho) \cup (g_s(\gamma) \cap g_s(\varrho))] \cap g_s(\varrho) = g_s(\varrho) \cap g_s(\varrho) = g_s(\varrho).
\end{aligned}
$$

$$
\begin{aligned}
&[(g_s \circ S_\varrho \circ g_s \circ S_\varrho \circ g_s) \tilde{\cap} g_s](\gamma) = \Big[\Big((g_s \circ S_\varrho)(\varrho) \cap (g_s \circ S_\varrho \circ g_s)(\gamma)\Big) \cup \Big((g_s \circ S_\varrho)(\gamma) \cap \\
&(g_s \circ S_\varrho \circ g_s)(\varrho)\Big)\Big] \cap g_s(\gamma) = \Big[\Big(g_s(\varrho) \cap \big(g_s(\varrho) \cap g_s(\gamma)\big)\Big) \cup \Big(g_s(\gamma) \cap \big((g_s(\varrho) \cup g_s(\gamma)\big)\Big)\Big] \cap \\
&g_s(\gamma) = [(g_s(\varrho) \cap g_s(\gamma)) \cup g_s(\gamma)] \cap g_s(\gamma) = g_s(\gamma) \cap g_s(\gamma) = g_s(\gamma).
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
&[(g_s \circ S_\gamma \circ g_s \circ S_\gamma \circ g_s) \tilde{\cap} g_s](\varrho) = [((g_s \circ S_\gamma)(\varrho) \cap (g_s \circ S_\gamma \circ g_s)(\varrho)) \cup ((g_s \circ S_\gamma)(\gamma) \cap \\
&(g_s \circ S_\gamma \circ g_s)(\gamma))] \cap g_s(\varrho) = \Big[\Big(g_s(\gamma) \cap \big(g_s(\varrho) \cap g_s(\gamma)\big)\Big) \cup \Big(g_s(\varrho) \cap \big(g_s(\varrho) \cup g_s(\gamma)\big)\Big)\Big] \cap \\
&g_s(\varrho) = [(g_s(\varrho) \cap g_s(\gamma)) \cup g_s(\varrho)] \cap g_s(\varrho) = g_s(\varrho) \cap g_s(\varrho) = g_s(\varrho).
\end{aligned}
$$

$[(g_s \circ S_\gamma \circ g_s \circ S_\gamma \circ g_s) \tilde{\cap} g_s](\gamma) = [((g_s \circ S_\gamma)(\wp) \cap (g_s \circ S_\gamma \circ g_s)(\gamma)) \cup ((g_s \circ S_\gamma)(\gamma) \cap (g_s \circ S_\gamma \circ g_s)(\wp))] \cap g_s(\gamma) = [(g_s(\gamma) \cap (g_s(\wp) \cup g_s(\gamma))) \cup (g_s(\wp) \cap (g_s(\wp) \cap g_s(\gamma)))] \cap g_s(\gamma) = [g_s(\gamma) \cup (g_s(\wp) \cap g_s(\gamma))] \cap g_s(\gamma) = g_s(\gamma) \cap g_s(\gamma) = g_s(\gamma)$.

Therefore, for all $x \in S$, $(g_s \circ S_x \circ g_s \circ S_x \circ g_s) \tilde{\cap} g_s \neq \emptyset_s$. Thereby, g_s is an SI-weakly almost BQI-ideal.

Proposition 2. If $\varkappa_S$ is a nonnull SI-BQI-ideal, then $\varkappa_S$ is an SI-almost BQI-ideal.

Proof: Suppose that $\varkappa_S \neq \emptyset_S$ is an SI-BQI-ideal, then $\varkappa_S \circ \tilde{\mathbb{S}} \circ \varkappa_S \circ \tilde{\mathbb{S}} \circ \varkappa_S \tilde{\subseteq} \varkappa_S$. Since $\varkappa_S \neq \emptyset_S$, by *Corollary 2*, it follows that $\varkappa_S \circ S_x \circ \varkappa_S \circ S_y \circ \varkappa_S \neq \emptyset_S$. We should show that

$$(\varkappa_S \circ S_x \circ \varkappa_S \circ S_y \circ \varkappa_S) \tilde{\cap} \varkappa_S \neq \emptyset_S.$$

for all $x, y \in S$. Since $\varkappa_S \circ S_x \circ \varkappa_S \circ S_y \circ \varkappa_S \tilde{\subseteq} \varkappa_S \circ \tilde{\mathbb{S}} \circ \varkappa_S \circ \tilde{\mathbb{S}} \circ \varkappa_S \tilde{\subseteq} \varkappa_S$, it follows that $\varkappa_S \circ S_x \circ \varkappa_S \circ S_y \circ \varkappa_S \tilde{\subseteq} \varkappa_S$. Thus,

$$(\varkappa_S \circ S_x \circ \varkappa_S \circ S_y \circ \varkappa_S) \tilde{\cap} \varkappa_S = \varkappa_S \circ S_x \circ \varkappa_S \circ S_y \circ \varkappa_S \neq \emptyset_S.$$

implying that t_S is an SI-almost BQI-ideal.

The soft set $\emptyset_S$ is an SI-BQI-ideal, as $\emptyset_S \circ \tilde{\mathbb{S}} \circ \emptyset_S \circ \tilde{\mathbb{S}} \circ \emptyset_S \tilde{\subseteq} \emptyset_S$; but it is not SI-almost BQI-ideal, since $(\emptyset_S \circ S_x \circ \emptyset_S \circ S_y \circ \emptyset_S) \tilde{\cap} \emptyset_S = \emptyset_S$, for all $x, y \in S$.

Proposition 2 shows that SI-almost BQI-ideal is a generalization of nonnull SI-BQI-ideal. Here, note that if f_S is an SI-almost-BQI-ideal, f_S need not be an SI-BQI-ideal, as shown in the following example:

Example 3. In *Example 1*, f_S and l_S are SI-almost BQI-ideals; however, f_S and l_S are not SI-BQI-ideals. In fact;

$$\begin{aligned}(f_S \circ \tilde{\mathbb{S}} \circ f_S \circ \tilde{\mathbb{S}} \circ f_S)(\wp) &= [(f_S \circ \tilde{\mathbb{S}})(\wp) \cap (f_S \circ \tilde{\mathbb{S}} \circ f_S)(\wp)] \cup [(f_S \circ \tilde{\mathbb{S}})(\gamma) \cap (f_S \circ \tilde{\mathbb{S}} \circ f_S)(\gamma)]\\ &= [(f_S(\wp) \cup f_S(\gamma)) \cap (f_S(\wp) \cup f_S(\gamma))] \cup [(f_S(\wp) \cup f_S(\gamma)) \cap (f_S(\wp) \cup f_S(\gamma))]\\ &= f_S(\wp) \cup f_S(\gamma) \not\subseteq f_S(\wp).\end{aligned}$$

Thus, f_s is not an SI-BQI-ideal. Similarly,

$$\begin{aligned}(l_S \circ \tilde{\mathbb{S}} \circ l_S \circ \tilde{\mathbb{S}} \circ l_S)(\wp) &= [(l_S \circ \tilde{\mathbb{S}})(\wp) \cap (l_S \circ \tilde{\mathbb{S}} \circ l_S)(\wp)] \cup [(l_S \circ \tilde{\mathbb{S}})(\gamma) \cap (l_S \circ \tilde{\mathbb{S}} \circ l_S)(\gamma)]\\ &= [(l_S(\wp) \cup l_S(\gamma)) \cap (l_S(\wp) \cup l_S(\gamma))] \cup [(l_S(\wp) \cup l_S(\gamma)) \cap (l_S(\wp) \cup l_S(\gamma))]\\ &= l_S(\wp) \cup l_S(\gamma) \not\subseteq l_S(\wp).\end{aligned}$$

Thus, l_s is not an SI-BQI-ideal.

Proposition 3. Let $\varkappa_S$ be an idempotent SI-almost BQI-ideal. Then, $\varkappa_S$ is an SI-almost subsemigroup.

Proof: assume that $\varkappa_S$ is an idempotent SI-almost BQI-ideal. Then, $\varkappa_S \circ \varkappa_S = \varkappa_S$ and $(\varkappa_S \circ S_x \circ \varkappa_S \circ S_y \circ \varkappa_S) \tilde{\cap} \varkappa_S \neq \emptyset_S$, for all $x, y \in S$. We should show that

$$(\varkappa_S \circ \varkappa_S) \tilde{\cap} \varkappa_S \neq \emptyset_S.$$

for all $x \in S$. Since,

$$\begin{aligned}\emptyset_S \neq (\varkappa_S \circ S_x \circ \varkappa_S \circ S_y \circ \varkappa_S) \tilde{\cap} \varkappa_S &= [(\varkappa_S \circ S_x \circ \varkappa_S \circ S_y \circ \varkappa_S) \tilde{\cap} \varkappa_S] \tilde{\cap} \varkappa_S\\ &= [(\varkappa_S \circ S_x \circ \varkappa_S \circ S_y \circ \varkappa_S) \tilde{\cap} (\varkappa_S \circ \varkappa_S)] \tilde{\cap} \varkappa_S \tilde{\subseteq} (\varkappa_S \circ \varkappa_S) \tilde{\cap} \varkappa_S.\end{aligned}$$

Hence, $(\varkappa_S \circ \varkappa_S) \tilde{\cap} \varkappa_S \neq \emptyset_S$, implying that $\varkappa_S$ is an SI-almost subsemigroup.

Proposition 3 shows that an idempotent SI-almost subsemigroup is a generalization of SI-almost BQI-ideal.

Theorem 4. Let $f_S \tilde{\subseteq} q_S$ such that f_S is an SI-almost BQI-ideal. Then, q_S is an SI-almost BQI-ideal.

Proof: suppose that f_S is an SI-almost BQI-ideal. Hence, $(f_S \circ S_x \circ f_S \circ S_y \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, for all $x, y \in S$. We should show that $(q_S \circ S_x \circ q_S \circ S_y \circ q_S) \tilde{\cap} q_S \neq \emptyset_S$. In fact,

$$(f_S \circ S_x \circ f_S \circ S_y \circ f_S) \tilde{\cap} f_S \tilde{\subseteq} (q_S \circ S_x \circ q_S \circ S_y \circ q_S) \tilde{\cap} q_S$$

Since $(f_S \circ S_x \circ f_S \circ S_y \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, thus $(q_S \circ S_x \circ q_S \circ S_y \circ q_S) \tilde{\cap} q_S \neq \emptyset_S$.

Theorem 5. Let f_S and q_S be SI-almost BQI-ideals. Then, $f_S \tilde{\cup} q_S$ is an SI-almost BQI-ideal.

Proof: let f_S and q_S be SI-almost BQI-ideals. Since $f_S \tilde{\subseteq} f_S \tilde{\cup} q_S$, $f_S \tilde{\cup} q_S$ is an SI-almost BQI-ideal by *Theorem 4*.

Note that if f_S and q_S are SI-almost BQI-ideals, then $f_S \tilde{\cap} q_S$ need not be an SI-almost BQI-ideal.

Example 4. Consider the SI-(weakly) almost BQI-ideals f_S and l_S in *Example 1*.

$$f_S \tilde{\cap} l_S = \{(\wp, \emptyset), (\Upsilon, \emptyset)\} = \emptyset_S.$$

Thus, $f_S \tilde{\cap} l_S$ is not an SI-almost BQI-ideal.

Theorem 5 and *Example 4* show that the union of SI-almost BQI-ideals is an SI-almost BQI-ideal, but the intersection of SI-almost BQI-ideals need not be an SI-almost BQI-ideal. This is distinct from the classical ideal theory of semigroup.

Lemma 2. Let $x \in S$ and $\emptyset \neq X \subseteq S$. Then, $S_x \circ S_Y = S_{xY}$. Similarly, if $\emptyset \neq X \subseteq S$ and $y \in S$, then $S_X \circ S_y = S_{Xy}$ [50].

Theorem 6. Let $\emptyset \neq \mathcal{H} \subseteq S$. Then, $\mathcal{H}$ is an almost BQI-ideal if and only if $S_{\mathcal{H}}$, the soft characteristic function of $\mathcal{H}$, is an SI-almost BQI-ideal.

Proof: let $\emptyset \neq \mathcal{H}$ is an almost BQI-ideal. Then, $\mathcal{H}x\mathcal{H}y\mathcal{H} \cap \mathcal{H} \neq \emptyset$, for all $x, y \in S$. Thus, there exists $t \in S$ such that $t \in \mathcal{H}x\mathcal{H}y\mathcal{H} \cap \mathcal{H}$. Then,

$$\left((S_{\mathcal{H}} \circ S_x \circ S_{\mathcal{H}} \circ S_y \circ S_{\mathcal{H}}) \tilde{\cap} S_{\mathcal{H}}\right)(t) = (S_{\mathcal{H}x\mathcal{H}y\mathcal{H}} \tilde{\cap} S_{\mathcal{H}})(t) = (S_{\mathcal{H}x\mathcal{H}y\mathcal{H} \cap \mathcal{H}})(t) = U \neq \emptyset.$$

implying that $(S_{\mathcal{H}} \circ S_x \circ S_{\mathcal{H}} \circ S_y \circ S_{\mathcal{H}}) \tilde{\cap} S_{\mathcal{H}} \neq \emptyset_S$. Therefore, $S_{\mathcal{H}}$ is an SI-almost BQI-ideal.

Conversely, let $S_{\mathcal{H}}$ be an SI-almost BQI-ideal. Thus, $(S_{\mathcal{H}} \circ S_x \circ S_{\mathcal{H}} \circ S_y \circ S_{\mathcal{H}}) \tilde{\cap} S_{\mathcal{H}} \neq \emptyset_S$, for all $x, y \in S$. To show that $\mathcal{H}$ is an almost BQI-ideal of S, we should prove that and $\mathcal{H}x\mathcal{H}y\mathcal{H} \cap \mathcal{H} \neq \emptyset$, for all $x, y \in S$. Now

$$\begin{aligned}
&\emptyset_S \neq (S_{\mathcal{H}} \circ S_x \circ S_{\mathcal{H}} \circ S_y \circ S_{\mathcal{H}}) \tilde{\cap} S_{\mathcal{H}} \Rightarrow \text{ there exists } n \in \\
&S\,; \left((S_{\mathcal{H}} \circ S_x \circ S_{\mathcal{H}} \circ S_y \circ S_{\mathcal{H}}) \tilde{\cap} S_{\mathcal{H}}\right)(n) \neq \emptyset, \\
&\Rightarrow \text{there exists } n \in S;\ \left((S_{\mathcal{H}x\mathcal{H}y\mathcal{H}}) \tilde{\cap} S_A\right)(n) \neq \emptyset, \\
&\Rightarrow \text{there exists } n \in S;\ (S_{\mathcal{H}x\mathcal{H}y\mathcal{H} \cap \mathcal{H}})(n) \neq \emptyset, \\
&\Rightarrow \text{there exists } n \in S;\ (S_{\mathcal{H}x\mathcal{H}y\mathcal{H} \cap \mathcal{H}})(n) = U, \\
&\Rightarrow n \in \mathcal{H}x\mathcal{H}y\mathcal{H} \cap \mathcal{H}.
\end{aligned}$$

Hence, $\mathcal{H}x\mathcal{H}y\mathcal{H} \cap \mathcal{H} \neq \emptyset$, for all $x, y \in S$. Accordingly, $\mathcal{H}$ is an almost BQI-ideal.

Theorem 6 explains to us that if we have a set that we know is almost BQI-ideal, then its soft characteristic function is an SI-almost BQI-ideal, and vice versa. *Theorem 6* is of great importance as it serves as a bridge between classical semigroup theory and soft set theory.

Example 5. Let S=$\{\wp, \Upsilon, \aleph\}$ be given with the following *Table 2*.

Table 2. Cayley table of the binary operation.

.	ϱ	ɤ	ჱ
ϱ	ϱ	ɤ	ჱ
ɤ	ɤ	ɤ	ჱ
ჱ	ɤ	ɤ	ჱ

Then $(S,.)$ is a semigroup. In [46], it was shown that $\mathcal{H}=\{$ϱ, ჱ$\}$ is an almost BQI-ideal. By routine calculations, one can show that

$$S_{\mathcal{H}}=\{(\text{ϱ}, U),(\text{ɤ}, \emptyset),(\text{ჱ}, U)\}.$$

is an SI-almost BQI-ideal. Conversely, by choosing the soft characteristic function of X as $S_X=\{(\text{ϱ}, U),(\text{ɤ}, \emptyset),(\text{ჱ}, U)\}$, by routine calculations, one can show that S_X is an SI-almost BQI-ideal and $X=\{$ϱ, ჱ$\}$ is an almost BQI-ideal.

Lemma 3. Let t_S be a soft set over U. Then, $t_S \mathrel{\tilde{\subseteq}} S_{\operatorname{supp}(t_S)}$ [50].

Theorem 7. If t_S is an SI-almost BQI-ideal, then $\operatorname{supp}(t_S)$ is an almost BQI-ideal.

Proof: let t_S be an SI-almost BQI-ideal. Thus, $t_S \neq \emptyset_S$, thus $\operatorname{supp}(t_S) \neq \emptyset$. Moreover, $(t_S \circ S_x \circ t_S \circ S_y \circ t_S) \mathrel{\tilde{\cap}} t_S \neq \emptyset_S$, for all $x, y \in S$. To show that $\operatorname{supp}(t_S)$ is an almost BQI-ideal; by *Theorem 6*, we should show that $S_{\operatorname{supp}(t_S)}$ is an SI-almost BQI-ideal. By *Lemma 3*,

$$(t_S \circ S_x \circ t_S \circ S_y \circ t_S) \mathrel{\tilde{\cap}} t_S \mathrel{\tilde{\subseteq}} (S_{\operatorname{supp}(t_S)} \circ S_x \circ S_{\operatorname{supp}(t_S)} \circ S_y \circ S_{\operatorname{supp}(t_S)}) \mathrel{\tilde{\cap}} S_{\operatorname{supp}(t_S)}.$$

And since $(t_S \circ S_x \circ t_S \circ S_y \circ t_S) \mathrel{\tilde{\cap}} t_S \neq \emptyset_S$, it implies that

$$(S_{\operatorname{supp}(t_S)} \circ S_x \circ S_{\operatorname{supp}(t_S)} \circ S_y \circ S_{\operatorname{supp}(t_S)}) \mathrel{\tilde{\cap}} S_{\operatorname{supp}(t_S)} \neq \emptyset_S.$$

Accordingly, $S_{\operatorname{supp}(t_S)}$ is an SI-almost BQI-ideal of S, and $\operatorname{supp}(t_S)$ is an almost BQI-ideal by *Theorem 6*.

Theorem 7 explains to us that if we have a soft set that we know that it is an SI-almost BQI-ideal, then its support is an almost BQI-ideal, and vice versa. *Theorem 7* is of great importance as it serves as a bridge between classical semigroup theory and soft set theory as well. Here, note that if $\operatorname{supp}(t_S)$ is an almost BQI-ideal, then f_S need not be an SI-almost BQI-ideal, as shown in the following example:

Example 6. In *Example 1*, it is shown that g_S is not an SI-almost BQI-ideal. It is obvious that $\operatorname{supp}(g_S)=\{$ϱ, ɤ$\}$. Since,

[supp(g_S){ϱ}supp(g_S){ϱ}supp(g_S)] ∩ supp(g_S) = {ϱ, ɤ}ϱ{ϱ, ɤ}ϱ{ϱ, ɤ} ∩ {ϱ, ɤ} = {ϱ, ɤ}.
[supp(g_S){ϱ}supp(g_S){ɤ}supp(g_S)] ∩ supp(g_S) = {ϱ, ɤ}ϱ{ϱ, ɤ}ɤ{ϱ, ɤ} ∩ {ϱ, ɤ} = {ϱ, ɤ}.
[supp(g_S){ɤ}supp(g_S){ϱ}supp(g_S)] ∩ supp(g_S) = {ϱ, ɤ}ɤ{ϱ, ɤ}ϱ{ϱ, ɤ} ∩ {ϱ, ɤ} = {ϱ, ɤ}.
[supp(g_S){ɤ}supp(g_S){ɤ}supp(g_S)] ∩ supp(g_S) = {ϱ, ɤ}ɤ{ϱ, ɤ}ɤ{ϱ, ɤ} ∩ {ϱ, ɤ} = {ϱ, ɤ}.

It follows that $[\operatorname{supp}(g_S)\{x\}\operatorname{supp}(g_S)\{y\}\operatorname{supp}(g_S)] \cap \operatorname{supp}(g_S) \neq \emptyset$, for all $x, y \in S$. Thereby, $\operatorname{supp}(g_S)$ is an almost BQI-ideal of S.

Definition 12. An SI-almost BQI-ideal f_S is called minimal if for any SI-almost BQI-ideal y_S whenever $y_S \mathrel{\tilde{\subseteq}} f_S$, then $\operatorname{supp}(y_S)=\operatorname{supp}(f_S)$.

Theorem 8. Let $\mathcal{H}$ be a nonempty subset of S. Then, $\mathcal{H}$ is a minimal almost BQI-ideal if and only if $S_{\mathcal{H}}$ is a minimal SI-almost BQI-ideal.

Proof: let $\mathcal{H}$ be a minimal almost BQI-ideal. Thus, $\mathcal{H}$ is an almost BQI-ideal of S, and so $S_{\mathcal{H}}$ is an SI-almost BQI-ideal by *Theorem 6*. Let f_S be an SI-almost BQI-ideal such that $f_S \tilde{\subseteq} S_{\mathcal{H}}$. By *Theorem 7*, $\operatorname{supp}(f_S)$ is an almost BQI-ideal, and by *Note 1* and *Corollary 2*.

$$\operatorname{supp}(f_S) \subseteq \operatorname{supp}(S_{\mathcal{H}}) = \mathcal{H}.$$

Since $\mathcal{H}$ is a minimal almost BQI-ideal, $\operatorname{supp}(f_S) = \operatorname{supp}(S_{\mathcal{H}}) = \mathcal{H}$. Thereby, $S_{\mathcal{H}}$ is a minimal SI-almost BQI-ideal by *Definition 12*.

Conversely, let $S_{\mathcal{H}}$ be a minimal SI-almost BQI-ideal. Thus, $S_{\mathcal{H}}$ is an SI-almost BQI-ideal of S, and $\mathcal{H}$ is an almost BQI-ideal by *Theorem 6*. Let $\mathcal{M}$ be an almost BQI-ideal such that $\mathcal{M} \subseteq \mathcal{H}$. By *Theorem 6*, $S_{\mathcal{M}}$ is an SI-almost BQI-ideal, and by *Theorem 2*, $S_{\mathcal{M}} \tilde{\subseteq} S_{\mathcal{H}}$. As $S_{\mathcal{H}}$ is a minimal SI-almost BQI-ideal,

$$\mathcal{M} = \operatorname{supp}(S_{\mathcal{M}}) = \operatorname{supp}(S_{\mathcal{H}}) = \mathcal{H}.$$

by *Corollary 1*. Thus, $\mathcal{H}$ is a minimal almost BQI-ideal.

Theorem 8 shows us the relation between minimal almost BQI-ideal and its soft characteristic function, and it demonstrates that if a nonempty set is a minimal almost BQI-ideal, then its soft characteristic function is a minimal SI-almost BQI-ideal, and vice versa.

Definition 13. Let f_S, g_S, and y_S be any SI-almost BQI-ideals. If $y_S \circ g_S \tilde{\subseteq} f_S$ implies that $y_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-prime almost BQI-ideal.

Definition 14. Let f_S and y_S be any SI-almost BQI-ideals. If $y_S \circ y_S \tilde{\subseteq} f_S$ implies that $y_S \tilde{\subseteq} f_S$, then f_S is called an SI-semiprime almost BQI-ideal.

Definition 15. Let f_S, g_S, and y_S be any SI-almost BQI-ideals. If $(y_S \circ g_S) \tilde{\cap} (g_S \circ y_S) \tilde{\subseteq} f_S$ implies that $y_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-strongly prime almost BQI-ideal.

Every SI-strongly prime almost BQI-ideal of S is an SI-prime almost BQI-ideal, and every SI-prime almost BQI-ideal of S is an SI-semiprime almost BQI-ideal is obvious.

Theorem 9. If $S_{\wp}$ is an SI-prime almost BQI-ideal, then $\wp$ is a prime almost BQI-ideal, where $\emptyset \neq \wp \subseteq S$.

Proof: let $S_{\wp}$ be an SI-prime almost BQI-ideal. Thus, $S_{\wp}$ is an SI-almost BQI-ideal of S and, thus, $\wp$ is an almost BQI-ideal by *Theorem 6*. Let $\mathcal{H}$ and $\mathcal{M}$ be almost BQI-ideals such that $\mathcal{H}\mathcal{M} \subseteq P$. Thus, by *Theorem 6*, $S_{\mathcal{H}}$ and $S_{\mathcal{M}}$ are SI-almost BQI-ideals, and by *Theorem 2,*

$$S_{\mathcal{H}} \circ S_{\mathcal{M}} = S_{\mathcal{H}\mathcal{M}} \tilde{\subseteq} S_{\wp}.$$

Since $S_{\wp}$ is an SI-prime almost BQI-ideal and $S_{\mathcal{H}} \circ S_{\mathcal{M}} \tilde{\subseteq} S_{\wp}$, we obtain $S_{\mathcal{H}} \tilde{\subseteq} S_{\wp}$ or $S_{\mathcal{M}} \tilde{\subseteq} S_{\wp}$. Thereby, by *Theorem 2*, $\mathcal{H} \subseteq \wp$ or $\mathcal{M} \subseteq \wp$. Accordingly, $\wp$ is a prime almost BQI-ideal.

Theorem 10. If $S_{\wp}$ is an SI-semiprime almost BQI-ideal of S, then $\wp$ is a semiprime almost BQI-ideal, where $\emptyset \neq \wp \subseteq S$.

Proof: assume that $S_{\wp}$ is an SI-semiprime almost BQI-ideal. Thus, $S_{\wp}$ is an SI-almost BQI-ideal and $\wp$ is an almost BQI-ideal of S by *Theorem 6*. Let $\mathcal{H}$ be an almost BQI-ideal such that $\mathcal{H}\mathcal{H} \subseteq \wp$. Thus, by *Theorem 6*, $S_{\mathcal{H}}$ is an SI-almost BQI-ideal, and by *Theorem 2*,

$$S_{\mathcal{H}} \circ S_{\mathcal{H}} = S_{\mathcal{H}\mathcal{H}} \tilde{\subseteq} S_{\wp}.$$

Since $S_{\wp}$ is an SI-semiprime almost BQI-ideal of S and $S_{\mathcal{H}} \circ S_{\mathcal{H}} \tilde{\subseteq} S_{\mathcal{H}}, S_{\mathcal{H}} \tilde{\subseteq} S_{\wp}$ is obtained. Thereby, by *Theorem 2*, $\mathcal{H} \subseteq \wp$. Accordingly, $\wp$ is a semiprime almost BQI-ideal.

Theorem 11. If $S_{\wp}$ is an SI-strongly prime almost BQI-ideal, then $\wp$ is a strongly prime almost BQI-ideal of S, where $\emptyset \neq \wp \subseteq S$.

Proof: assume that $S_{\wp}$ is an SI-strongly prime almost BQI-ideal. Thus, $S_{\wp}$ is an SI-almost BQI-ideal of S and, thus, $\wp$ is an almost BQI-ideal by *Theorem 6*. Let $\mathcal{H}$ and $\mathcal{M}$ be almost BQI-ideals such that $\mathcal{H}\mathcal{M} \cap \mathcal{M}\mathcal{H} \subseteq \wp$. Thus, by *Theorem 6*, $S_{\mathcal{H}}$ and $S_{\mathcal{M}}$ are SI-almost BQI-ideals, and by *Theorem 2*,

$$(S_{\mathcal{H}} \circ S_{\mathcal{M}}) \tilde{\cap} (S_{\mathcal{M}} \circ S_{\mathcal{H}}) = S_{\mathcal{H}\mathcal{M}} \tilde{\cap} S_{\mathcal{M}\mathcal{H}} = S_{\mathcal{H}\mathcal{M}\cap\mathcal{M}\mathcal{H}} \tilde{\subseteq} S_{\wp}.$$

Since $S_{\wp}$ is an SI-strongly prime almost BQI-ideal and $(S_{\mathcal{H}} \circ S_{\mathcal{M}}) \tilde{\cap} (S_{\mathcal{M}} \circ S_{\mathcal{H}}) \tilde{\subseteq} S_{\wp}$, we derive $S_{\mathcal{H}} \tilde{\subseteq} S_{\wp}$ or $S_{\mathcal{M}} \tilde{\subseteq} S_{\wp}$. Hence, by *Theorem 2*, $\mathcal{H} \subseteq \wp$ or $\mathcal{M} \subseteq \wp$. Hence, $\wp$ is a strongly prime almost BQI-ideal.

Theorem 9, *Theorem 10*, and *Theorem 11* construct the relation of the set and its characteristic function as regards certain types of priminess.

4 | Conclusion

Semigroup ideals and soft intersection ideals of semigroups are relevant to a wide range of fields of study, including commutative algebra and graph theory; therefore, many researchers have explored their generalization and produced a variety of results. As discussed in the introduction section, some of these generalizations provide insight into our work. We aim to formulate a more general notion that generalizes the concept of soft intersection BQI-ideals. We carry out this by introducing soft intersection (weakly) almost BQI-ideal, studying in detail the characteristics of the mentioned ideal, and investigating how they relate to other soft intersection almost ideal classes and ideals of semigroups.

Soft intersection almost BQI-ideal, as our analysis shows, is a generalization of soft intersection BQI-ideal, and soft intersection weakly almost BQI-ideal is a generalization of soft intersection almost BQI-ideal. It is important to remember, nevertheless, that the soft set must be nonnull in order for soft intersection BQI-ideal to be regarded as soft intersection almost BQI-ideal. We also demonstrated that, nevertheless, with the counterexamples, the converses are not true. We have answered questions about the algebraic structures of soft intersections that are almost BQI-ideals and found similarities as well as distinctions between soft intersections that are almost BQI-ideals and BQI-ideals by studying them. We have deduced that the soft union binary operation constructs a semigroup with the collection of soft intersection almost BQI-ideals, but the soft intersection operation does not. We have also obtained the relation between soft intersection minimal (reps., prime, semiprime, and strongly prime) almost BQI-ideal of a semigroup and minimal (reps., prime, semiprime, and strongly prime) almost BQI-ideal of a semigroup according to with the derived key theorem that if a nonempty set of a semigroup is an almost BQI-ideal, then its soft characteristic function is a soft intersection almost BQI-ideal, and vice versa. We also manage to establish a relation between classical semigroup theory and soft set theory with this vital theorem.

As a result, this work has brought up several open-ended issues for further investigation. To inspire the reader, we include a few of these below. Investigating many kinds of ideals, such as quasi-ideal, interior ideal, bi-ideal, bi-interior ideal, and bi-quasi ideal of semigroups and their interrelations, may provide important information. Readers with an interest in this field could find the studies regarding soft intersection almost ideal to be inspiring. Since when dealing with uncertainty, parametric approaches like soft sets work incredibly well, and new perspectives on the solution of parametric data issues may be provided by introducing new soft algebraic structures and determining their algebraic properties and implementations, it would be interesting to investigate to create new soft set-based category theory, graph applications, topology, cryptography or decision-making techniques for use in practical settings, business, or technology by considering these new soft algebraic structures. Furthermore, bipolar soft sets, lattice-ordered soft sets, and double-framed soft sets can all be handled with this new structure. An interesting line of inquiry is to use soft intersection BQI-ideals to characterize various forms of semigroups, such as regular, intraregular, and weakly regular semigroups.

Acknowledgments

This paper is derived from the second author's Master Thesis supervised by the first author at Amasya University, Türkiye.

Author Contribution

Conceptualization, A.S., and Z.H.B.; Methodology, A. S.; Software, A.İ.; Validation, A.S., Z.H.B., and A.İ.; formal analysis, A.S.; investigation, Z.H.B.; resources, A.S.; data maintenance, A.S.; writing-creating the initial design, A.İ.; writing-reviewing and editing, A.S.; visualization, A.İ.; monitoring, A.İ. All authors have read and agreed to the published version of the manuscript.

Funding

The authors declare that no funds, grants, or other support were received during the preparation of this manuscript.

Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflicts of Interest

The authors declare no conflict of interest concerning the reported research findings.

References

[1] Good, R. A., & Hughes, D. R. (1952). Associated groups for a semigroup. *Bulletin of the american mathematical society*, *58*(6), 624–625. https://www.researchgate.net/publication/292301495_Associated_groups_for_a_semigroup

[2] Steinfeld, O. (1956). Uher die quasi ideals, von halbgruppend. *Publicationes mathematicae debrecen*, *3–4*(23), 262–275. DOI: 10.5486/PMD.1956

[3] Grošek, O., & Satko, L. (1980). A new notion in the theory of semigroup. *Semigroup forum*, *20*(1), 233–240. DOI: 10.1007/BF02572683

[4] Bogdanovic, S. (1981). Semigroups in which some bi-ideal is a group. *Review of research faculty of science-university of novi sad*, *11*, 261–266. https://sites.dmi.uns.ac.rs/nsjom/Papers/11/NSJOM_11_261_266.pdf

[5] Wattanatripop, K., Chinram, R., & Changphas, T. (2018). Quasi-A-ideals and fuzzy A-ideals in semigroups. *Journal of discrete mathematical sciences and cryptography*, *21*(5), 1131–1138. DOI: 10.1080/09720529.2018.1468608

[6] Kaopusek, N., Kaewnoi, T., & Chinram, R. (2020). On almost interior ideals and weakly almost interior ideals of semigroups. *Journal of discrete mathematical sciences and cryptography*, *23*(3), 773–778. DOI: 10.1080/09720529.2019.1696917

[7] Iampan, A., Chinram, R., & Petchkaew, P. (2021). A note on almost subsemigroups of semigroups. *International journal of mathematics and computer science*, *16*(4), 1623–1629. http://ijmcs.future-in-tech.net

[8] Chinram, R., & Nakkhasen, W. (2022). Almost bi-quasi-interior ideals and fuzzy almost bi-quasi-interior ideals of semigroups. *Journal of mathematics and computer science*, *26*(2), 128–136. DOI: 10.22436/jmcs.026.02.03

[9] Gaketem, T. (2022). Almost bi interior ideal in semigroups and their fuzzifications. *European journal of pure and applied mathematics*, *15*(1), 281–289. DOI: 10.29020/nybg.ejpam.v15i1.4279

[10] Gaketem, T., & Chinram, R. (2023). Almost bi-quasi-ideals and their fuzzifications in semigroups. *Annals of the university of craiova, mathematics and computer science series*, *50*(2), 342–352. DOI: 10.52846/ami.v50i2.1708

[11] Wattanatripop, K., Chinram, R., & Changphas, T. (2018). Fuzzy almost bi-ideals in semigroups. *International journal of mathematics and computer science*, *13*(1), 51–58. http://ijmcs.future-in-tech.net/13.1/R-2Chinram.pdf

[12] Krailoet, W., Simuen, A., Chinram, R., & Petchkaew, P. (2021). A note on fuzzy almost interior ideals in semigroups. *International journal of mathematics and computer science*, *16*(2), 803–808. https://l1nq.com/FPikj

[13] Molodtsov, D. (1999). Soft set theory - first results. *Computers and mathematics with applications*, *37*(4–5), 19–31. DOI: 10.1016/s0898-1221(99)00056-5

[14] Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers and mathematics with applications*, *45*(4–5), 555–562. DOI: 10.1016/S0898-1221(03)00016-6

[15] Pei, D., & Miao, D. (2005, July). From soft sets to information systems. *2005 IEEE international conference on granular computing* (pp. 617-621). IEEE. DOI: 10.1109/GRC.2005.1547365

[16] Sezgin, A., Aybek, F. N., & Güngör, N. B. (2023). New soft set operation: complementary soft binary piecewise union operation. *Acta informatica malaysia*, *7*(1), 38–53. DOI: 10.26480/aim.01.2023.38.53

[17] Sezgin, A., & Demirci, A. M. (2023). A new soft set operation: complementary soft binary piecewise star (*) operation. *Ikonion journal of mathematics*, *5*(2), 24–52. DOI: 10.54286/ikjm.1304566

[18] Sezgin, A., Ahmad, S., & Mehmood, A. (2019). A new operation on soft sets: extended difference of soft sets. *Journal of new theory*, (27), 33–42. https://dergipark.org.tr/en/pub/jnt/issue/43609/535589

[19] Sezgin, A., & Cagman, N. (2024). A new soft set operation: complementary soft binary piecewise difference (\) operation tt - yeni bir esnek küme işlemi: tümleyenli esnek ikili parçalı fark (\) Işlemi. *Osmaniye korkut ata üniversitesi fen bilimleri enstitüsü dergisi*, *7*(1), 58–94. DOI: 10.47495/okufbed.1308379

[20] Sezgin, A., & Yavuz, E. (2023). A new soft set operation: soft binary piecewise symmetric difference operation. *Necmettin erbakan üniversitesi fen ve mühendislik bilimleri dergisi*, *5*(2), 189–208. DOI: 10.47112/neufmbd.2023.18

[21] Sezgin, A., & Demirci, A. M. (2023). A new soft set operation: complementary soft binary piecewise star (*) operation. *Ikonion journal of mathematics*, *5*(2), 24–52. DOI: 10.54286/ikjm.1304566

[22] Ali, M. I., Feng, F., Liu, X., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. *Computers and mathematics with applications*, *57*(9), 1547–1553. DOI: 10.1016/j.camwa.2008.11.009

[23] Sezgin, A., & Atagün, A. O. (2011). On operations of soft sets. *Computers and mathematics with applications*, *61*(5), 1457–1467. DOI: 10.1016/j.camwa.2011.01.018

[24] Feng, F., Jun, Y. B., & Zhao, X. (2008). Soft semirings. *Computers and mathematics with applications*, *56*(10), 2621–2628. DOI: 10.1016/j.camwa.2008.05.011

[25] Ali, M. I., Shabir, M., & Naz, M. (2011). Algebraic structures of soft sets associated with new operations. *Computers and mathematics with applications*, *61*(9), 2647–2654. DOI: 10.1016/j.camwa.2011.03.011

[26] Stojanović, N. (2021). A new operation on soft sets: 791 extended symmetric difference of soft sets. *Vojnotehnicki glasnik*, *69*(4), 779–791. DOI: 10.5937/vojtehg69-33655

[27] SEZGİN, A., & Atagün, A. (2023). A new soft set operation: complementary soft binary piecewise plus (+) operation. *Matrix science mathematic*, *7*(2), 125–142. DOI: 10.26480/msmk.02.2023.125.142

[28] Sezgin, A., & Yavuz, E. (2023). A new soft set operation: complementary soft binary piecewise lamda (λ) operation tt - yeni bir esnek küme işlemi: tümleyenli esnek ikili parçalı lambda (λ) işlemi. *Sinop üniversitesi fen bilimleri dergisi*, *8*(2), 101–133. DOI: 10.33484/sinopfbd.1320420

[29] Sezgin, A., Aybek, F. N., & Atagün, A. O. (2023). A new soft set operation: complementary soft binary piecewise intersection (∩) operation. *Black sea journal of engineering and science*, *6*(4), 330–346. DOI: 10.34248/bsengineering.1319873

[30] Çağman, N., & Enginoğlu, S. (2010). Soft set theory and uni–int decision making. *European journal of operational research*, *207*(2), 848–855. DOI: 10.1016/j.ejor.2010.05.004

[31] Çağman, N., Çitak, F., & Aktaş, H. (2012). Soft int-group and its applications to group theory. *Neural computing and applications*, *21*(1), 151–158. DOI: 10.1007/s00521-011-0752-x

[32] Sezer, A. S., Agman, N., Atagün, A. O., Ali, M. I., & Turkmen, E. (2015). Soft intersection semigroups, ideals and bi-ideals: a new application on semigroup theory I. *Filomat*, *29*(5), 917–946. DOI: 10.2298/FIL1505917S

[33] Sezer, A. S., Çağman, N., & Atagün, A. O. (2014). Soft intersection interior ideals, quasi-ideals and generalized bi-ideals; A new approach to semigroup theory II. *Journal of multiple-valued logic and soft computing*, *23*(1–2), 161–207. https://acesse.dev/JoyEj

[34] Sezgin, A., & Orbay, M. (2022). Analysis of semigroups with soft intersection ideals. *Acta universitatis sapientiae, mathematica*, *14*(1), 166–210. DOI: 10.2478/ausm-2022-0012

[35] Mahmood, T., Rehman, Z. U., & Sezgin, A. (2018). Lattice ordered soft near rings. *Korean journal of mathematics*, *26*(3), 503–517. DOI: 10.11568/kjm.2018.26.3.503

[36] Jana, C., Pal, M., Karaaslan, F., & Sezgin, A. (2019). (α, β)-soft intersectional rings and ideals with their applications. *New mathematics and natural computation*, *15*(2), 333–350. DOI: 10.1142/S1793005719500182

[37] Mustuoglu, E., Sezgin, A., & Kaya, Z. (2016). Some characterizations on soft uni-groups and normal soft uni-groups. *International journal of computer applications*, *155*(10), 1–8. DOI: 10.5120/ijca2016912412

[38] Sezer, A. S. (2014). Certain characterizations of LA-semigroups by soft sets. *Journal of intelligent and fuzzy systems*, *27*(2), 1035–1046. DOI: 10.3233/IFS-131064

[39] Sezer, A. S., Çağman, N., & Atagün, A. O. (2015). Uni-soft substructures of groups. *Annals of fuzzy mathematics and informatics*, *9*(2), 235-246. http://www.afmi.or.kr/papers/2015/Vol-09_No-02/PDF/AFMI-9-2(235-246)-H-140701R2.pdf

[40] Özlü, S., & Sezgin, A. (2020). Soft covered ideals in semigroups. *Acta universitatis sapientiae, mathematica*, *12*(2), 317–346. DOI: 10.2478/ausm-2020-0023

[41] Atagün, A. O., & Sezgin, A. (2018). Soft subnear-rings, soft ideals and soft n-subgroups of near-rings. *Mathematical sciences letters*, *7*(1), 37–42. DOI: 10.18576/msl/070106

[42] Sezgin, A. (2018). A new view on AG-groupoid theory via soft sets for uncertainty modeling. *Filomat*, *32*(8), 2995–3030. DOI: 10.2298/FIL1808995S

[43] Sezgin, A., Çağman, N., & Atagün, A. O. (2017). A completely new view to soft intersection rings via soft uni-int product. *Applied soft computing journal*, *54*, 366–392. DOI: 10.1016/j.asoc.2016.10.004

[44] Sezgin, A., Atagün, A. O., Çağman, N., & Demir, H. (2022). On near-rings with soft union ideals and applications. *New mathematics and natural computation*, *18*(2), 495–511. DOI: 10.1142/S1793005722500247

[45] Rao, M. M. K. (2018). Bi-interior ideals of semigroups. *Discussiones mathematicae - general algebra and applications*, *38*(1), 69. DOI: 10.7151/dmgaa.1283

[46] Murali, K. (2018). A study of a generalization of bi-ideal, quasi ideal and interior ideal of semigroup. *Mathematica moravica*, *22*(2), 103–115. DOI: 10.5937/matmor1802103m

[47] Rao, M. M. K. (2020). Left bi-quasi ideals of semigroups. *Southeast asian bulletin of mathematics*, *44*(3), 369–376. https://l1nq.com/4gadf

[48] Rao, M. M. K. (2020). Quasi-interior ideals and weak-interior ideals. *Asia pacific journal of mathematics*, *7*, 7–21. DOI: 10.28924/APJM/7-21

[49] Baupradist, S., Chemat, B., Palanivel, K., & Chinram, R. (2021). Essential ideals and essential fuzzy ideals in semigroups. *Journal of discrete mathematical sciences and cryptography*, *24*(1), 223–233. DOI: 10.1080/09720529.2020.1816643

[50] Sezgin, A., & İlgin, A. (2024). Soft intersection almost subsemigroups of semigroups. *International journal of mathematics and physics*, *15*(1), 13–20. DOI: 10.26577/ijmph.2024v15i1a2

[51] Sezgin, A., & İlgin, A. (2024). Soft intersection almost ideals of semigroups TT - yarıgrupların esnek kesişimsel hemen hemen idealleri. *Journal of innovative engineering and natural science*, *4*(2), 466–481. DOI: 10.61112/jiens.1464344

[52] Pant, S., Dagtoros, K., Kholil, M. I., & Vivas, A. (2024). Matrices: peculiar determinant property. *Optimum science journal*, (1), 1–7. DOI: 10.5281/zenodo.11266018

[53] Sezgin, A., & Çalişici, H. (2024). A comprehensive study on soft binary piecewise difference operation. *Eskişehir teknik üniversitesi bilim ve teknoloji dergisi b - teorik bilimler*, *12*(1), 32–54. DOI: 10.20290/estubtdb.1356881

[54] Sezgin, A., & Sarialioğlu, M. (2024). Yeni bir esnek küme işlemi: tümleyenli esnek ikili parçalı theta (θ) işlemi. *Kadirli uygulamalı bilimler fakültesi dergisi*, *4*(2), 325–357. https://kadirliubfd.com/index.php/kubfd/article/view/97

[55] Mostafa, N., Krishan Kumar, A., & Ali, Y. (2024). A comparative study on x-ray image enhancement based on Neutrosophic set. *Sustainable machine intelligence journal*, *7*, (2):1-10. DOI: 10.61356/smij.2024.77102

[56] El-Morsy, S. (2023). Stock portfolio optimization using pythagorean fuzzy numbers. *Journal of operational and strategic analytics*, *1*(1), 8–13. DOI: 10.56578/josa010102

[57] Adak, A. K., Kumar, D., & Edalatpanah, S. A. (2024). Some new operations on pythagorean fuzzy sets. *Uncertainty discourse and applications*, *1*(1), 11–19. https://www.uda.reapress.com/journal/article/view/17

[58] A Comparative Study of Hybrid Systems Using RAMD Analysis. (2024). *Risk assessment and management decisions*, *1*(1), 75–87. https://www.ramd.reapress.com/journal/article/view/26

[59] Fakhraoui, M., Bakhadach, I., Achik, S., & Sadiki, H. (2024). Pythagorean fuzzy artinian and Noetherian ring. *Computational algorithms and numerical dimensions*, *3*(1), 73–78. DOI: 10.22105/cand.2024.459526.1096

[60] Rahman, A. U. (2023). A theoretical context for (θ,β)-convexity and (θ,β)-concavity with hypersoft settings. *Big data and computing visions*, *3*(4), 196–208. DOI:10.22105/bdcv.2023.192676

[61] Khalifa, H. A, Edalatpanah, S. A., & Bozanic, D. (2024). On min-max goal programming approach for solving piecewise quadratic fuzzy multi- objective de novo programming problems. *Systemic analytics*, *2*(1), 35–47. DOI: 10.31181/sa21202411

[62] Muniz, S. M., Andriola, W. B., Fátima Muniz, R., & Edalatpanah, S. A. (2023). The impacts of Covid-19 on postgraduate courses at the federal university of ceará (UFC, Brazil). *Optimality*, *1*(1), 1–9. DOI: 10.22105/opt.v1i1.18

[63] Edalatpanah, S. A., Khalifa, H. A. E. W., & Keyser, R. S. (2024). A new approach for solving fuzzy cooperative continuous static games. *Soft computing fusion with applications*, *1*(1 SE-Articles), 19–26. https://scfa.reapress.com/journal/article/view/19